Chapter 2

Reactors in Series
1. **Introduction**

There are a number of practical reasons for using two or more small reactors rather than one large reactor. Temperature control is better in smaller reactors and interstage cooling can be used. Sometimes the shape of the curve in the Levenspiel plot dictates that minimizing the reactor volume requires two or more reactors. It is often advantage to mix and match reactors.

1.1. **CSTRs in Series**

When CSTRs are placed in series the equation for the first reactor is

\[ V_{CSTR} = \frac{F_A X_1}{(-r_A)} \]  \hspace{1cm} (1)

The equation for the second and subsequent CSTRs is modified somewhat

\[ V_{CSTR} = \frac{F_A (X_f - X_1)}{(-r_A)} \]  \hspace{1cm} (2)
The equation for the first reactor can be written as

\[ V_{CSTR} = \frac{F_{A0} \ (X_1 - X_0)}{(-r_A)} \]  

(3)

but, it is usually abbreviated to Equation ?? because \( X_0 \) is most often equal to zero. Looking at the Levenspiel plot for a isothermal reaction, the areas for two reactors in series are represented by the two rectangles shown on the plot. The enclosed white space is proportional to the extra volume that would be needed to achieve the same conversion with one reactor.

Figure 1: Levenspiel plot showing the areas that are proportional to the volumes for \( CSTR_1 \) – Cyan, \( CSTR_2 \) – Blue, and the volume of one reactor to do the same conversion.
1.2. Plug Flow Reactors

The equations for the PFRs in series are

\[ V_{PFR_1} = F_{A0} \int_{X_0}^{X_1} \frac{1}{(-r_A)} dX \]  \hspace{1cm} (4)

\[ V_{PFR_2} = F_{A0} \int_{X_1}^{X_f} \frac{1}{(-r_A)} dX \]  \hspace{1cm} (5)

PFRs in series are the same as one big PFR whose volume is equal to the sum of the volume of all of the smaller PFRs. This can be seen by inspection of a Levenspiel plot.
Figure 2: Two PFRs in series. \( PFR_1 \) – Cyan, \( PFR_2 \) – Blue.

The first PFR converts the reactant stream to 0.4 and the second PFR takes the output of the first PFR and converts the stream to 0.65. This is equivalent to using one large PFR that takes the input stream and produces the same final conversion (Equation ??).

\[
F_A0 \int_0^{0.4} \frac{1}{(-r_A)} \, dX + F_A0 \int_{0.4}^{0.65} \frac{f}{(-r_A)} \, dX = F_A0 \int_0^{0.65} \frac{1}{(-r_A)} \, dX \quad (6)
\]

This result is not a function of the shape of the curve in the Levenspiel plot.