Example: \( G_p(s) = \frac{2e^{-0.2s}}{(s+1)} \)

* Desired closed-loop response: \( G_{cl}(s) = \frac{e^{-\Theta s}}{\lambda s + 1} \) \( \iff \) \( G_{cc} \) must include exponential term.

* Controller equation:

\[
G_c(s) = \frac{G_{cl}(s)}{G_p(s)[1 - G_{cl}(s)]} = \frac{e^{-\Theta s}}{2e^{-0.2s}} \left[ \frac{1}{(\lambda s + 1)} - \frac{e^{-\Theta s}}{(\lambda s + 1)} \right] = \frac{e^{-0.2s}(s+1)}{2e^{-0.2s}(\lambda s + 1) - e^{-\Theta s}}
\]

\[
G_c(s) = \frac{0.5e^{-(\Theta-0.2)s}}{(\lambda s + 1) - e^{-\Theta s}} \quad \{ Taylor \; series \; expansion: \}
\]

\[
G_c(s) = \frac{0.5e^{-(\Theta-0.2)s}}{\lambda s + 1 - (1 - \Theta s)} = \frac{0.5e^{-(\Theta-0.2)s}(s+1)}{(\lambda + \Theta)s}
\]

**PT controller:** \( K_c \frac{\tau_I s + 1}{\tau_I s} \)

For our example, \( \tau_I = 1.0 \)

\[
K_c = \frac{0.5}{\lambda + \Theta}
\]

\{ If \( \Theta_{cl} = \Theta_p \) the exponential term will vanish. Otherwise \( \Theta_{cl} \) must be greater than \( \Theta_p \), so that the exponent is 0. \}
Example: \[ g_p(s) = \frac{-9s+1}{(15s+1)(3s+1)} \]

Previous Approach: \[ g_{cl}(s) = \frac{1}{\lambda s+1} \rightarrow g_c(s) = \frac{(15s+1)(3s+1)}{(-9s+1)\lambda s} \]

New Approach: \[ g_{cl}(s) = \frac{-9s+1}{(\lambda s+1)^2} \rightarrow g_c(s) = \frac{(15s+1)(3s+1)}{(-9s+1)(\lambda s+1)^2} \]

\[ g_c(s) = \frac{-9s+1}{(\lambda s+1)^2} \left[ 1 - \left( \frac{-9s+1}{(\lambda s+1)^2} \right) \right] = \frac{(15s+1)(3s+1)}{[(\lambda s+1)^2 - (-9s+1)]} \]

\[ g_c(s) = \frac{(45s^2 + 18s + 1)}{[\lambda^2 s^2 + 22s + 1 + 9s - 1]} = \left[ \frac{(45s^2 + 18s + 1)}{(18s)} \right] \left( \frac{\frac{\lambda^2}{18} s + \frac{\lambda}{9} + \frac{1}{2}}{1} \right) \]

\[ g_c(s) = \frac{(45s^2 + 18s + 1)}{(18s)} \cdot \frac{1}{[\lambda^2 s^2 + 22s + 1 + 9s - 1]} \]

\[ g_c(s) = \left( \frac{18}{2\lambda + 9} \right) \cdot \left( \frac{45s^2 + 18s + 1}{18s} \right) \cdot \left( \frac{1}{\lambda^2 \cdot s + (2\lambda + 9)/18} \right) \]

Constant PID controller 1st order filter

\[ g_c(s) = K_c \frac{\tau_0 \tau_i s^2 + \tau_d s + 1}{\tau_0 s} \]

PID controller: \[ g_c(s) = K_c \left( \tau_0 \tau_i s^2 + \tau_d s + 1 \right) \]

\[ K_c = \frac{18}{2\lambda + 9} \quad \tau_i = 18 \quad \tau_d = 2.5 \]
Example (§6.10)

\[ G_p(s) = \frac{15}{s^2 + 4.9s + 0.9} \]

* Use the direct synthesis method, specifying 1st order closed-loop behavior:

\[ G_{cl}(s) = \frac{K}{(s+1)} \]

* Find the PID tuning parameters if a closed-loop time constant of 5 minutes is desired.

\[ G_{cl}(s) = \frac{1}{5s + 1} \]

\[ G_c(s) = \frac{G_{cl}(s)}{G_p(s)[1 - G_{cl}(s)]} \]

\[ G_{oc}(s) = \frac{1}{5s + 1} \left( \frac{15}{s^2 + 4.9s + 0.9} \right) = \frac{s^2 + 4.9s + 0.9}{15(5s+1)(1 - \frac{1}{5s+1})} = \frac{75s}{s^2 + 4.9s + 0.9} \]

\[ G_{PID}(s) = K_c \left( \frac{\tau_D s + \tau_I s + 1}{\tau_I s} \right) \]

\[ G_c(s) = \frac{10}{9} s^2 + \frac{49}{9} s + 1 \left( \frac{10}{9} \right) \frac{1}{75s} = \frac{10}{9} s^2 + \frac{49}{9} s + 1 \left( \frac{10}{9} \right) 75s \]

\[ K_c = 0.06533 \]
\[ \tau_I = 5.444 \]
\[ \tau_D = \frac{10}{49} = 0.20408 \]
\[ P = 0.06533 \]
\[ I = 0.01200 \]
\[ D = 0.01333 \]
Controller Tuning: modern industrial plants typically include thousands of individual control loops. After a controller is installed, the preliminary settings are usually satisfactory. However, for some critical control loops, the preliminary settings may have to be adjusted.

A) Controller tuning involves a tradeoff between performance & robustness.

B) Controller settings do not have to be precisely determined (+/- 10%).

C) For most plants, it is not feasible to tune each controller.

- only the control loops that are the most important or most troublesome receive detailed attention.

Guidelines for Common Control Loops: useful for situations where a process model is not available.

Flow Rate: flow control loops commonly used in the chemical process industries. About 50% of the control loops in oil refineries are used for flow control. Flow and pressure control loops have fast responses, with essentially no time delay. Disturbances tend to be frequent but relatively small. Most are high frequency noise due to turbulence, valve changes, or pump vibrations.

→ PI control is typically used, with settings of: 0.5 < Kc < 0.7

→ High frequency noise discourages the use of derivative action b/c it amplifies the noise.

→ Because flow control loops have small settling times, there is little incentive to use derivative action (to make the loop respond even faster).
**Liquid Level:** Standard P or PI controllers are used for level control. Integral control action is often used but can be omitted if small offsets in the liquid level (±5%) can be tolerated. Derivative action is not normally used because the level measurements are often noisy.

\[ K_c = \frac{100\%}{\Delta h} \]

\[ T_i = \frac{4V}{K_c \cdot Q_{\text{max}}} \]

\[ \Delta h = \min (h_{\text{max}} - h_{\text{sp}}, h_{\text{sp}} - h_{\text{min}}) \]

\( V = \text{tank volume} \)

\( Q_{\text{max}} = \text{maximum flow rate} \)

For some applications, tight level control is needed: some chemical reactors, bioreactors.

**Gas Pressure:** High and low limits are usually a serious concern for pressure control. For self-regulating processes, pressure is relatively easy to control, except when the gas is in equilibrium with a liquid. For pressure control, PI controllers are normally used with only a small amount of integral action. Usually, the vessel is not large, leading to small residence times. Derivative action is not usually needed because the response times are usually small.

**Temperature:** Guidelines are difficult to state for the wide variety of processes and operations: heat exchangers, distillation columns, chemical reactors, evaporators, etc. Time delays and varying thermal capacity usually place a stability limit on \( K_c \). PID controllers are often used to provide more rapid responses.

**Composition:** Similar to temperature control loops, with some differences:

1) Measurement noise is a significant problem.
2) Time delays with analyzer may be significant.