Module 3b: Flow in Pipes

Darcy-Weisbach

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Darcy-Weisbach Equation

- Based upon theory.
- Used to estimate energy loss due to friction in pipe.

$$h_f = f \frac{L}{D} \left( \frac{V^2}{2g} \right)$$

Where
- $h_f$ = head loss (feet)
- $L$ = length of pipe (feet)
- $f$ = friction factor
- $D$ = internal diameter of pipe (feet)
- $V^2/2g$ = velocity head (feet)

Darcy-Weisbach Equation

- Darcy-Weisbach can be written for flow (substitute $V = Q/A$, where $A = (\pi/4)D^2$ in the above equation):

$$h_f = f \frac{8L}{\pi^2D^3} \left( \frac{Q^2}{g} \right)$$

- Darcy-Weisbach can be rewritten to solve for velocity:

$$V = \left( \frac{h_f \times 2g \times D}{L \times f} \right)^{0.5}$$

Darcy-Weisbach Equation

Friction factor for Darcy-Weisbach Equation:

- Based upon the Reynolds number, $N_R$ or $Re$, and a dimensionless parameter called the relative roughness, $\varepsilon/d$ or $\varepsilon/d$ ($\varepsilon$ = absolute roughness; $d$ = diameter).

- For laminar flow:

$$f = \frac{64}{N_R} = \frac{64}{Re}$$

- For turbulent flow, friction factor must be read off a Moody diagram (or off a relative roughness vs. friction factor diagram for completely turbulent flow).
Example:
- A polymeric coagulant, undiluted, has an absolute viscosity of 0.48 kg/(m·sec) [0.01 lb-sec/ft²] and a specific gravity of 1.15. This fluid is to be pumped at the rate of 3.78 L/min (1 gallon/min) through 15.25 m (50 ft) of ½-inch diameter schedule 40 pipe (ID = 0.622 in = 15.8 mm = 0.0158 m). What is the head loss due to friction?

To use Darcy-Weisbach to calculate head loss, need $f$:

$$ h_f = f rac{L}{D} \left( \frac{V^2}{2g} \right) $$

Darcy-Weisbach Equation

If the flow is laminar, the friction factor can be calculated. Otherwise, it must be looked up off the chart.

Need to determine velocity in order to calculate Reynolds number (determine if flow is laminar or turbulent).

By Continuity Equation:

$$ Q = VA \quad \text{or} \quad V = \frac{Q}{A} $$

Substituting:

$$ V = \frac{(3.78 \text{ L/min})(1 \text{ m}^3/1000 \text{ L})(1 \text{ min}/60 \text{ sec})}{(\pi/4)(0.0158 \text{ m})^2} $$

$$ V = 0.32 \text{ m/sec} \quad \text{or} \quad V = 0.32 \text{ m/sec} $$
**Darcy-Weisbach Equation**

• Calculate the Reynolds number to see if flow is laminar or turbulent.

\[
Re = \frac{VD\rho}{\mu}
\]

• Substituting:

Definition of Specific Gravity = Fluid Density/Water Density

Density of Water = \( \rho_{H2O} = 1000 \text{ kg/m}^3 \)

\[
Re = \frac{(0.321 \text{ m/sec})(0.0158 \text{ m})(1.15)(1000 \text{ kg/m}^3)}{(0.48 \text{ kg/m – sec})}
\]

\[Re = 12.15\]

**Darcy-Weisbach Equation**

• HAVE LAMINAR FLOW!!

• Therefore, the friction factor for Darcy-Weisbach is calculated as follows:

\[
f = \frac{64}{Re}
\]

\[f = \frac{64}{12.15}
\]

\[f = 5.27\]

**Example:**

– A 24-inch class I ductile iron pipe (ID = 0.63 m = 24.79 in.) 90 m long with a neat cement lining (asphalted cast iron) carries a flow of water at 1.5 m³/sec (52.9 ft³/sec). What is the friction loss in the pipe?

\[
h_f = \frac{(5.27)(15.25 \text{ m})(0.321 \text{ m/sec})^2}{(0.0158 \text{ m})(2)(9.806 \text{ m/sec}^2)}
\]

\[h_f = 26.7 m\]

• Friction Slope:

\[
\text{Friction Slope} = \frac{26.7 m}{15.25 m} = 1.75 = 175\%
\]
Darcy-Weisbach Equation

- Calculate Reynolds number.
  Assuming that the fluid is water: \( v = 1.003 \times 10^{-6} \text{ m}^2/\text{sec} \) at 20\(^\circ\)C.
  - Find velocity of flow.
    \[ v = \frac{Q}{A} = \frac{1.5 \text{ m}^3/\text{sec}}{\frac{\pi}{4} \times (0.63 \text{ m})^2} \]
    \[ V = 4.81 \text{ m/sec} \]
  - Find the Reynolds number.
    \[ \text{Re} = \frac{VD}{\nu} = \frac{(4.81 \text{ m/sec})(0.63 \text{ m})}{(1.003 \times 10^{-6} \text{ m}^2/\text{sec})} \]
    \[ \text{Re} = 3.02 \times 10^6 \]

Darcy-Weisbach Equation

- Reading from the Moody Diagram:
  \( f(\epsilon/d) = 0.0002 \) & \( \text{Re} = 3.02 \times 10^6 \) = 0.014

- Substituting into Darcy-Weisbach:
  \( h_f = \frac{(0.014)(90 \text{ m})(4.81 \text{ m/sec})^3}{(0.63 \text{ m})(2)(9.806 \text{ m/sec}^2)} \]
  \[ h_f = 2.36 \text{ m} \]

Moody Diagram

Darcy-Weisbach Equation

- Example:
  - Determine the flowrate in a 500-m section of a 1-m diameter commercial steel pipe when there is a 2-m drop in the energy grade line over the section.
  Given: \( L = 500 \text{ m} \)
  \( D = 1 \text{ m} \)
  Commercial steel pipe
  \( h_f = 2 \text{ m} \)
  - Want to use Darcy-Weisbach equation with flow rate.
  \[ h_f = f \frac{8L}{\pi^2D^5} \left( \frac{Q^2}{g} \right) \]
Darcy-Weisbach Equation

• Need to find the friction factor.
  – As a first assumption about the flow in the pipe, will assume fully turbulent flow. Using Moody diagram for relative roughness in turbulent flow:
    \[ f = 0.0105 \]

Darcy-Weisbach Equation

• Solve Darcy-Weisbach for flow rate:

\[
 h_f = f \frac{8L}{\pi D} \left( \frac{Q^2}{g} \right)
\]

\[
 Q^2 = \frac{h_f \pi^2 g D^3}{8 \mu L}
\]

Substituting:

\[
 Q = \left( \frac{2 \pi^2 (9.81 \text{m/sec}^2)(1 \text{m})^3}{8(0.0105 \text{X 500m})} \right)^{1/2}
\]

\[
 Q = 2.1472 \text{m}^3/\text{sec}
\]

Darcy-Weisbach Equation

• Must check assumption of fully turbulent flow (i.e., was the \( f \) selected from the figure acceptable?).

• Using the continuity equation:

\[
 Q = VA = \frac{\pi}{4} V D^2
\]

\[
 2.1472 \text{m}^3/\text{sec} = \frac{\pi}{4} V (1.0\text{m})^2
\]

\[
 V = 2.7339 \text{m/sec}
\]

Darcy-Weisbach Equation

• Calculate the Reynolds number. (Note that no temperature is given for the fluid. Assume the fluid is water and the temperature is 15°C).

At 15°C, \( \nu = 1.139 \times 10^{-6} \text{m}^2/\text{sec} \)

• Substituting:

\[
 \text{Re} = \frac{VD}{\nu}
\]

\[
 \text{Re} = \frac{(2.7339 \text{m/sec})(1.0\text{m})}{1.139 \times 10^{-6} \text{m}^2/\text{sec}}
\]

\[
 \text{Re} = 2.40 \times 10^6
\]
Darcy-Weisbach Equation

- In order to use the full Moody diagram, need the relative roughness. For commercial steel, \( \varepsilon = 0.000046 \text{ m} \).
- Calculate:
  \[
  \varepsilon/D = (0.000046 \text{ m})/(1.0 \text{ m}) = 0.000046
  \]

Using the full Moody diagram, for \( \text{Re} = 2.40 \times 10^6 \) and \( \varepsilon/D = 0.000046 \):

- \( f = 0.0115 \)

Substituting back into Darcy-Weisbach for flow:

\[
Q = \frac{h \pi^2 g D^5}{8 f L}
\]

Substituting:

\[
Q = \frac{(2m) \pi^2 (9.8 \text{ m/s}^2)(1 \text{ m})^5}{8(0.0115)(500 \text{ m})^{7/2}}
\]

\[
Q = 2.05 \text{ m}^3 / \text{sec}
\]

Are we done?

Darcy-Weisbach Equation

- By continuity:
  \[
  V = \frac{Q}{A} = \frac{2.05 \text{ m}^3 / \text{sec}}{4/4}(1.0 \text{ m})
  \]

\[
V = 2.6 \text{ m/s}
\]

- Calculate the Reynolds number associated with this velocity:

\[
\text{Re} = \frac{VD}{\nu} = \frac{(2.6 \text{ m/s})(1.0 \text{ m})}{1.139 \times 10^{-6} \text{ m}^2 / \text{sec}}
\]

\[
\text{Re} = 2.3 \times 10^6
\]
Using the full Moody diagram, for Re = 2.30 x 10^6 and ε/D = 0.000046:  

\[ f \approx 0.0115 \]

**Darcy-Weisbach Equation**

- Friction factors approximately the same between last two iterations. Can use value from previous iteration as Q.

\[ Q = 2.05 \text{ m}^3/\text{sec} \]

Friction factors approximately the same between last two iterations.

**Are we done?** Yes.

**Example:**
- A 14-inch schedule 80 pipe (commercial steel) has an inside diameter of 12.5 in (317.5 mm). How much flow can this pipe carry if the allowable head loss is 3.5 m in a length of 200 m?

- Need to solve for V and A to get Q, the flow.

**Darcy-Weisbach Equation**

- In order to use the full Moody diagram, need the relative roughness. For commercial steel, \( \varepsilon = 0.000046 \text{ m} \).

- Calculate:
  \( \varepsilon/D = 0.0046 \text{ mm}/317.5 \text{ mm} = 0.000014 \)
For this \( \varepsilon/d \), \( f \) ranges from 0.0087 to 0.04, depending on the flow (expressed as the Reynolds number). Assume \( f = 0.01 \) (in range near low end). If \( f = 0.01 \), then the Reynolds number is approximately \( 4 \times 10^6 \).

\[
Darcy-Weisbach Equation
\]

- Substituting this into Darcy-Weisbach:

\[
V = \frac{[h_f \times 2g \times D]^{0.5}}{[f \times L]^{0.5}}
\]

\[
V = \frac{[(3.5 \text{ m})(2)(9.806 \text{ m}/\text{sec}^2)(0.3175 \text{ m})]^{0.5}}{[(200 \text{ m})(0.01)]^{0.5}}
\]

\[
V = 3.30 \text{ m/sec}
\]

Are we done?

Darcy-Weisbach Equation

- Now need to check the friction factor assumption:
  - Calculate Reynolds number for \( V = 3.30 \text{ m/sec} \).

\[
Re = \frac{VD}{V}
\]

\[
Re = \frac{3.30 \text{ m/sec}(0.3175 \text{ m})}{1.003 \times 10^{-6} \text{ m}^2/\text{sec}}
\]

\[
Re = 1.04 \times 10^6
\]

From the Moody diagram, the \( f \) for this Reynolds number is 0.012.
Darcy-Weisbach Equation

- Recalculate using Darcy-Weisbach:

\[ V = \left( \frac{k_l \times 2 \times g \times D}{f \times L} \right)^{0.5} \]

\[ V = \frac{(3.5 \text{ m})(2)(9.806 \text{ m} / \text{sec}^2)(0.3175 \text{ m})}{(200 \text{ m})(0.012)} \]^{0.5}

\[ V = 3.01 \text{ m/sec} \]

Darcy-Weisbach Equation

Example:
- Determine the head loss in a 46-cm concrete pipe with an average velocity of 1.0 m/sec and a length of 30 m.

- Calculate \( \frac{\varepsilon}{d} \).
  \[ D = 46 \text{ cm} = 0.46 \text{ m} \]
  \[ \varepsilon = 0.001 \text{ ft} (0.3048 \text{ m/ft}) = 0.000305 \text{ mm} \]
  Therefore \( \frac{\varepsilon}{d} = \frac{0.0003 \text{ m}}{0.46 \text{ m}} = 0.00066 \]

Darcy-Weisbach Equation

- Calculate Reynolds number for \( V = 3.01 \text{ m/sec} \).

\[ \text{Re} = \frac{VD}{\nu} \]

\[ \text{Re} = \frac{(3.01 \text{ m/sec})(0.3175 \text{ m})}{1.003 \times 10^{-6} \text{ m}^2 / \text{sec}} \]

\[ \text{Re} = 0.95 \times 10^6 \]

CLOSE ENOUGH, So pipe will carry:

\[ Q = VA = (3.01 \text{ m/sec}) \left( \frac{\pi}{4} \right) (0.3175 \text{ m})^2 \]

\[ Q = 0.238 \text{ m}^3 / \text{sec} \]

Are we done? Yes.

Darcy-Weisbach Equation

- Calculate Reynolds number:

\[ \text{Re} = \frac{VD}{\nu} \]

\[ \text{Re} = \frac{(1 \text{ m/sec})(0.46 \text{ m})}{1.003 \times 10^{-6} \text{ m}^2 / \text{sec}} \]

\[ \text{Re} = 4.58 \times 10^5 \]
From the Moody diagram, the $f$ for this Reynolds number is 0.019.

Darcy-Weisbach Equation

- Substituting into Darcy-Weisbach:

$$h_t = f \frac{L}{D} \left( \frac{V^2}{2g} \right)$$

$$h_t = (0.019) \left( \frac{30m}{0.46m} \right) \left( \frac{(1m/sec)^2}{2(9.806m/sec^2)} \right)$$

$$h_t = 0.063m$$

Are we done? Yes.